

# UNIT-4

## Differential Kinematics

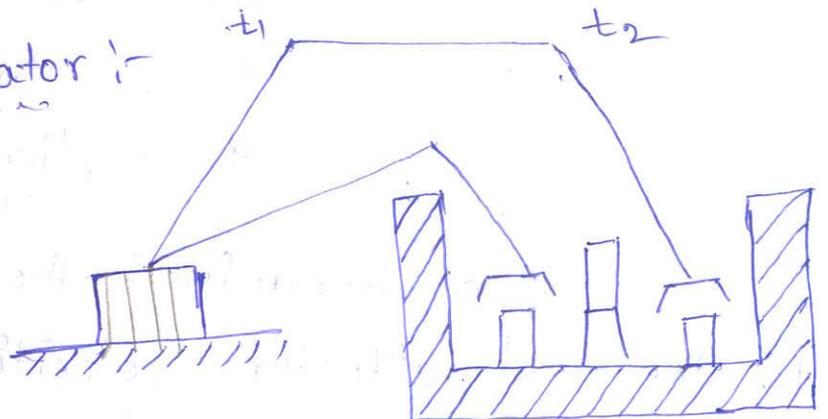
It is defined as various motions after manipulator that is joint velocities to the end effector velocities is called differential kinematics.

Planner Manipulator:- It is two links are rotating on one another of manipulator is called planner manipulator.

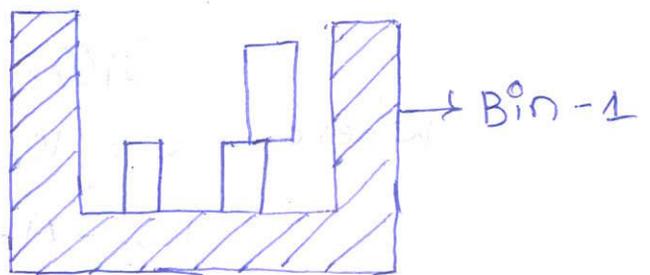
Spherical Manipulator:- A manipulator consisting of motion rotary and prismatic is turned as spherical manipulator.

Cartesian Space:- To define position and orientation of an object is a 3D space that is 3-displacement components and revolute components by the end effector is called "Cartesian Space."

Redundant Manipulator:-



Ex:- SCARA



A manipulator having more no. of degree of freedom than the required degree of freedom is called redundant manipulator.

In order to define the various position and orientation with the help of links there is a necessary of frame attached to the link - that's way means called the base frame.

Jacobian :-

The transformation matrix 'v' which desirable velocities from joint 1 to end factor matrix is called Jacobian.

Jacobian Manipulator

$$V_e(t) = J(q) \cdot \dot{q}$$

where  $\dot{q} = n$  joint velocities and

$V_e$  = velocity of end effectors

$J(q)$  = Jacobian matrix.

$t$  = Time factor

$q$  = displacement velocities.

To determines the various joints, velocities to the end effector velocities of manipulator mathematically expressed as

$$[J_1(q) \cdot J_2(q) \dots J_n(q)] \dot{q}$$

$$J_i(q) = \begin{bmatrix} v \\ \omega \end{bmatrix}$$

where  $v$  = linear velocity

$\omega$  = Angular velocity

$$\therefore J_i(q) = \begin{bmatrix} J_{uxi} \\ J_{wxi} \end{bmatrix}$$

$$J_i(q) = \begin{bmatrix} J_{uxi} \\ J_{uyi} \\ J_{uzi} \\ J_{wx_i} \\ J_{wyi} \\ J_{wzi} \end{bmatrix}$$

Jacobian Competition:-

1. The prismatic Jacobian joint
2. The rotary Jacobian joint.

To determine the jacobian there are two ways below.

$$1. J_i(q) = J_{ui} = \begin{bmatrix} P_{i-1} \\ 0 \end{bmatrix}$$

$$2. J_i(q) = J_{wi} = \begin{bmatrix} P_{i-1} \times {}^{i-1}P_n \\ P_{i-1} \end{bmatrix}$$

$$J_i(q) = \begin{bmatrix} P_{i-1} \\ 0 \end{bmatrix}_{P_{i-1}} = O_{P_{i-1}} \times \hat{u}$$

Where  $O_{P_{i-1}}$  is the rotation sub matrix and  $\hat{u}$  is the unit vector Acting along  $z_{i-1}$  axis

$${}^{i-1}P_n = O_{P_n} - O_{P_{i-1}}$$

$${}^{i-1}P_n = O_{T_n} O_n - O_{P_{i-1}} O_n$$

Where  $O_{Tn}$  = Transform matrix motion matrix from base frame to end effector matrix.

$O_n$  = The origin of the frame  $\{n\}$  that end effector.

$$O_n = [0 \ 0 \ 0 \ 1]^T$$

$$O_n = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$O_{T_{i-1}}$  = Identity matrix.

→ To determine the Jacobian matrix of column  $J_i$  is as follow 3 degree of freedom.

$$J^p(q) = \begin{bmatrix} P_{i-1} \times i-1 P_n \\ P_{i-1} \end{bmatrix}$$

$$i = 2, \quad n = 3.$$

$$P_{i-1} = P_{1-1} = P_0 = O_{R_{i-1}} \overset{n}{M}$$

$$P_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

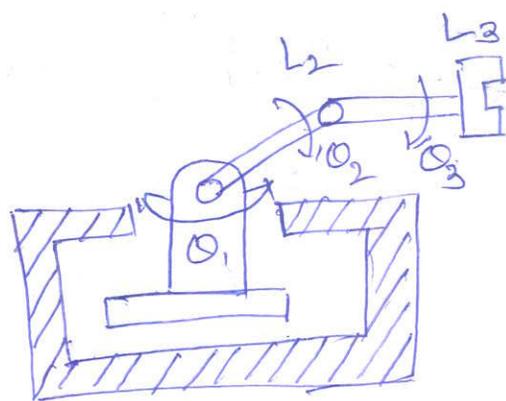
$$i-1 P_n = O_{Tn} O_n - O_{T_{i-1}} O_n$$

$$O_{P_3} = O_{T_3} \cdot O_n - O_{T_0} O_3$$

$$\boxed{i-1 P_n = O_{T_3} O_n - O_{T_0} O_n}$$

# Jacobian Problem

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$$\begin{aligned} \alpha_1 &= 90^\circ \\ \alpha_2 &= 0 \\ \alpha_3 &= 0 \end{aligned}$$

## Joint link parameters

Joint link	$a_i$	$d_i$	$\theta_i$	$\alpha_i$	$q_i$	$\cos \theta_i$	$\sin \theta_i$	$\cos \alpha_i$	$\sin \alpha_i$
1	0	0	$\theta_1$	$90$	$\theta_1$	$\cos \theta_1$	$\sin \theta_1$	0	1
2	$L_2$	0	$\theta_2$	0	$\theta_2$	$\cos \theta_2$	$\sin \theta_2$	1	0
3	$L_3$	0	$\theta_3$	0	$\theta_3$	$\cos \theta_3$	$\sin \theta_3$	1	0

$${}^0T_3 = {}^0T_1 \cdot {}^1T_2 \cdot {}^2T_3$$

$${}^i-1T_i = \begin{bmatrix} \cos \theta_i & -\sin \theta_i \cos \alpha_i & \sin \theta_i \sin \alpha_i & a_i \cos \theta_i \\ \sin \theta_i & \cos \theta_i \cos \alpha_i & -\cos \theta_i \sin \alpha_i & a_i \sin \theta_i \\ 0 & \sin \alpha_i & \cos \alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_1 = \begin{bmatrix} \cos \theta_1 & 0 & \sin \theta_1 & 0 \\ \sin \theta_1 & 0 & -\cos \theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1T_2 = \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 & 0 & L_2 \cos \theta_2 \\ \sin \theta_2 & \cos \theta_2 & 0 & L_2 \sin \theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2T_3 = \begin{bmatrix} \cos\alpha_3 & -\sin\alpha_3 & 0 & L_3 \cos\alpha_3 \\ \sin\alpha_3 & \cos\alpha_3 & 0 & L_3 \sin\alpha_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_3 = {}^0T_1 \cdot {}^1T_2 \cdot {}^2T_3$$

$${}^0T_3 = \begin{bmatrix} \cos\alpha_1 \cos\alpha_2 & -\cos\alpha_1 \sin\alpha_2 & \sin\alpha_1 & L_2 \cos\alpha_1 \cos\alpha_2 \\ \sin\alpha_1 \cos\alpha_2 & -\sin\alpha_1 \sin\alpha_2 & -\cos\alpha_1 & L_2 \sin\alpha_1 \cos\alpha_2 \\ \sin\alpha_2 & \cos\alpha_2 & 0 & L_2 \sin\alpha_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_3 = \begin{bmatrix} \cos\alpha_1 \cos\alpha_2 \cos\alpha_3 - \cos\alpha_1 \sin\alpha_2 \sin\alpha_3 & -\cos\alpha_1 \sin\alpha_2 \cos\alpha_3 - \cos\alpha_1 \cos\alpha_2 \sin\alpha_3 \\ \sin\alpha_1 \cos\alpha_2 \cos\alpha_3 - \sin\alpha_1 \sin\alpha_2 \sin\alpha_3 & -\sin\alpha_1 \cos\alpha_2 \cos\alpha_3 - \sin\alpha_1 \cos\alpha_2 \sin\alpha_3 \\ \sin\alpha_2 \cos\alpha_3 + \cos\alpha_2 \sin\alpha_3 & -\sin\alpha_2 \sin\alpha_3 + \cos\alpha_2 \cos\alpha_3 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \sin\alpha_1 & L_2 \cos\alpha_1 \cos\alpha_2 \cos\alpha_3 - L_3 \cos\alpha_1 \sin\alpha_2 \sin\alpha_3 + L_2 \cos\alpha_1 \cos\alpha_2 \\ -\cos\alpha_1 & L_2 \sin\alpha_1 \cos\alpha_2 \cos\alpha_3 - L_3 \sin\alpha_1 \cos\alpha_2 \sin\alpha_3 + L_2 \sin\alpha_1 \cos\alpha_2 \\ 0 & L_3 \cos\alpha_3 \sin\alpha_2 + L_3 \sin\alpha_3 \cos\alpha_2 + L_2 \sin\alpha_2 \\ 0 & 1 \end{bmatrix}$$

compare with end effector matrix

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$$\begin{bmatrix} n_x & o_x & a_x & d_x \\ n_y & o_y & a_y & d_y \\ n_z & o_z & a_z & d_z \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\rightarrow n_x = \cos\theta_1 \cos\theta_2 \cos\theta_3 - \sin\theta_2 \cos\theta_1 \sin\theta_3$$

$$n_y = \sin\theta_1 \cos\theta_2 \cos\theta_3 - \sin\theta_1 \sin\theta_2 \sin\theta_3$$

$$n_z = \sin\theta_2 \cos\theta_3 + \cos\theta_2 \sin\theta_3$$

$$\rightarrow o_x = -\cos\theta_1 \sin\theta_2 \cos\theta_3 - \cos\theta_1 \sin\theta_2 \sin\theta_3$$

$$o_y = -\sin\theta_1 \sin\theta_2 \cos\theta_3 - \sin\theta_1 \cos\theta_2 \sin\theta_3$$

$$o_z = -\sin\theta_2 \sin\theta_3 + \cos\theta_2 \cos\theta_3$$

$$\Rightarrow a_x = \sin\theta_1$$

$$a_y = -\cos\theta_1$$

$$a_z = 0$$

$$\rightarrow d_x = L_3 \cos\theta_1 \cos\theta_2 \cos\theta_3 - L_3 \cos\theta_1 \sin\theta_2 \sin\theta_3 + L_2 \cos\theta_1 \cos\theta_2$$

$$d_y = L_3 \sin\theta_1 \cos\theta_2 \cos\theta_3 - L_3 \sin\theta_1 \sin\theta_2 \sin\theta_3 + L_2 \sin\theta_1 \cos\theta_2$$

$$d_z = L_3 \sin\theta_2 \cos\theta_3 + L_3 \sin\theta_2 \sin\theta_3 + L_2 \sin\theta_2$$

